

MODERN MATHEMATICS
IN THE SECONDARY SCHOOL

by 741

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THE PROBLEM

Introduction

The mathematics progress of America's secondary schools have been and continue to be a matter of great concern to the people of this country. This concern has resulted in efforts to bring about change in secondary school mathematics programs which, during the first half of this century, have appeared to be relatively static.

Two events which have pointed out the need for new developments in secondary school mathematics programs are: (1) the race for space supremacy between the United States and Russia; and (2) the study of the secondary school as compiled by Dr. James B. Conant in his book, The American High School Today. Apparently these factors have been instrumental in encouraging American educators to develop a program of modern mathematics for the secondary school.

In regard to this program, new direction to the development of mathematical concepts, the organization of ideas, the provision of new instructional materials, and the methods of teaching secondary school students have emerged. For example, mathematicians have urged that certain basic principles be developed in the secondary school. Thus, what mathematics is, and what mathematics is considered important, are the decisions of mathematicians.

Thus, the problem to educators and teachers is clear. It is to determine what portion of this mathematics can be taught in the secondary school, to whom it can be taught, and the manner in which it can be taught.

Statement of the Problem

It was the purpose of this study to (1) discover what modern mathematics

is; (2) review selected modern mathematical teaching materials for secondary school mathematics programs; and (3) discover what implications modern mathematics holds for the secondary school's mathematics program and the secondary school mathematics teacher.

Limiting the Study

The study will be limited to the developments in modern mathematics which will generally be applicable to modern mathematics courses in grades 11 and/or 12 of the American secondary school. In addition, the study will be primarily concerned with modern mathematics content and its presentation in available textbooks, rather than treatment of methods and techniques for teaching these new developments of modern mathematics in the secondary school mathematics program.

With respect to the third purpose listed above, the study is limited to those implications which are concerned with capability and preparedness of both individual secondary school systems and individual secondary school mathematics teachers to incorporate and to teach, respectively, the modern mathematical developments.

Procedures Employed in the Study

To begin this study, a review of the pertinent literature contained in the Kansas State University library and five modern mathematics textbooks received from publishing companies was conducted. The investigation produced a considerable amount of information pertaining to modern mathematics in the secondary school which was organized into the following categories for presentation:

1. Development of mathematics at the secondary level in the American

educational system.

2. Development of mathematics in secondary schools during the decade 1950-60.
3. Instructional materials developed and written by national study groups devoted to modern mathematics in the secondary school.
4. A textbook devoted to modern mathematics written for the secondary school.

BACKGROUND OF MATHEMATICS IN THE SECONDARY SCHOOL

Before discussing recent developments in modern mathematics and the secondary mathematics program, it seems important that attention be directed to the background of mathematics in the secondary school. Scrutiny of this background is deemed important because it provides insights into the evolution, over past years, of mathematics in the curriculum of our American secondary school.

The earliest secondary school in America was the Latin Grammar School. Its curriculum offered the student no mathematics.¹

During the eighteenth century a new type of school, the academy, arose and was greatly influenced by practical and scientific ideas of society as well as society's classical and religious aspects. Its curriculum included mathematics which served a functional purpose reflecting the vocational aspects of the times. Mathematics was deemed important because it made the citizen more effective in meeting the demands of his environment.²

¹Kinney and Purdy, Teaching Mathematics in the Secondary Schools, Rinehart and Co., Inc., New York, 1958, p. 18.

²Ibid., p. 20.

At the end of the nineteenth century the high school was the main institution of secondary education. Mathematics held a large and important place in its curriculum. In fact, maximum allotment of time which could be spent on mathematics increased from four years in 1860 to four and one-half years in 1900 while the minimum requirements increased from one year to two years during the same period.¹

At the beginning of the twentieth century, stagnation set in the mathematics program of the American secondary school. The progress in such programs failed to change with the changing society. Hence, there was a steady decline in the percentage of high school students taking mathematics courses in the period from 1900 to 1935.²

It was during the above mentioned period that the National Council of Teachers of Mathematics was organized (1920). This organization, perhaps more than any other, has aided in the development and strengthening of the American secondary school mathematics program. For example, in one of its early studies published in 1923 the National Council of Teachers of Mathematics enumerated the aims of the mathematics program to be threefold:

- (1) Practical: Algebra, geometry, arithmetic have daily importance in the life of every individual.
- (2) Disciplinary: "... acquisition in precise form, of ideas (concepts) in terms of which quantitative thinking is done, the ability to think clearly in such terms, and mental habits and attitudes that make these concepts effective."

¹Loc. cit.

²H. R. Douglas, "Lets Face the Facts," The Mathematics Teacher, February 1937, pp. 56-62.

(3) Cultural: The mathematics program should provide for the appreciation of beauty, ideals of perfection, and appreciation of the power of mathematics.¹

"Integrating all the aims was the development of functional thinking which the committee considered should be the major and unifying thread in mathematical education!"²

The Joint Commission to Study the Place of Mathematics in Secondary Education published its report in 1940. This group made an attempt to devise a mathematics program which would bring together the dual strands of mathematics for general education and mathematics in the college preparatory sequence.³

Though there existed a concern for the mathematics program the pre-Pearl Harbor period exhibited the tendency of the American Secondary School to belittle the importance of mathematics. In contrast, however, the increasing complexity of expanding economic and social life continually requires a higher level of mathematical literacy for competency in one's daily activities.

In regard to the need for a higher level of mathematical literacy for competency in one's daily activities Campbell and Caswell have this to say:

More than ever we need a reorganizing of the curriculum in mathematics, which thus far has resisted educational intelligence. The dead hand of the past generations still dominates the content of mathematics courses. Little or no effort is made to relate mathematics to life needs

¹Kinney and Purdy, Teaching Mathematics in the Secondary School, Rinehart and Co., Inc., New York, 1958, p. 23.

²Loc. cit.

³Kinney and Purdy, op. cit., p. 26.

⁴Ibid, p. 41.

or even correlate it with the mathematical situations in science classes.¹

Evidently, this early retort did little to stir the American educators to strengthen the secondary school mathematics courses and to increase the numbers of students studying mathematics. More recently, Alexander and Saylor have rephrased the above mentioned statement for needed revision and action in mathematics as follows:

Leading members of these professions (scientific fields) and citizens concerned about the national welfare have criticized the high schools for not enrolling more pupils in courses in science and mathematics. In fact, the percentage of high schools offering courses and particularly advanced courses in these fields has declined during the period of the past twenty-five years, and the percentage of pupils taking such subjects has, of course, declined. Those who considered mathematics and science essential subjects in the high school curriculum have been vigorous in their demands that more emphasis in the high school be given to these fields.²

Thus, it is evident that mathematics has been and continues to be an area of concern in the secondary school of America. Today, mathematics is the fastest growing and most radically changing of all the sciences. All the major theories of the past years are still valid, and yet, new ideas are coming to the front. To those who have attempted to remain aware of the trends and the thinking in mathematics and the secondary school; it is apparent that the secondary school mathematics program is in need of revision and the addition of content from modern mathematics.

It is felt that the National Council of Teachers of Mathematics and committees formed from its membership have provided much in the manner of ideas, standards, and guidelines for individual secondary school systems in developing their mathematics programs. Their early leadership has set the

¹Campbell and Caswell, Readings in Curriculum Development, Houghton and Mifflin Co., New York, 1937, p. 752.

²Alexander and Saylor, Modern Secondary Education, Rinehart and Co., Inc., New York, 1959, p. 109.

present for present study groups to follow. It has also shown the need for revising the secondary school's mathematics program in line with the views of the contemporary mathematical and educational needs.

RELATED STUDIES

Since the end of the nineteenth century there have been numerous groups which have attempted to review the mathematics program of the secondary school and provide general guidelines to assist individual schools in improving their mathematics program. Lacking the support of individual schools the recommendations of these committees have failed to appreciably help the secondary school mathematics program.

A new awareness that the mathematics program is weak and in need of revision has resulted in formation of many more groups of mathematicians and educators whose purpose is to improve the secondary school's mathematics program. With regard to secondary school mathematics, the period from 1950 to 1960 has been characterized by continuous detailed studies which have taken positive actions to improve the secondary school mathematics instruction as well as the content of mathematics courses. These committees have attempted to integrate the traditional mathematics with topics from the modern mathematics. Their actions and recommendations have received both state and local support.

The University of Illinois Committee on School Mathematics

In 1951 the University of Illinois Committee on School Mathematics (UICSM) was established. It has since carried on a strong program of study and experimentation in the improvement of secondary school mathematics. This committee has done most of its work in the subject matter fields of algebra

and geometry.

In addition to the teaching materials developed by this group; they have cooperated with several universities in giving summer school training sessions to secondary school teachers who are or will be using their course outlines.

The Commission on Mathematics of the College
Entrance Examination Board

The Commission on Mathematics of the College Entrance Examination Board (CEEBS) was organized in 1955, with Dr. Albert Tucker of Princeton University as its chairman. The commission, composed of mathematicians and secondary school teachers, has set the guidelines for revision and modification, as well as making many recommendations for major improvements in the secondary school mathematics program. This group has periodically published the findings and recommendations derived from its continuous study.

The Commission has prepared sample textbooks in the areas of mathematics which will be new to the secondary school mathematics program. The purpose of these texts is to serve as a guide to those people who will be involved in writing or selecting such books for secondary school use.

The School Mathematics Study Group

A group of some one-hundred college mathematicians and secondary school teachers was organized under the name of the School Mathematics Study Group (SMSG) in 1958. This group was established by three major United States mathematical organizations (American Mathematical Society, the Mathematical Association, and the National Council of Teachers of Mathematics) and has as its aim the strengthening of mathematics instruction in secondary schools. The SMSG's project has been to collaborate in the planning and writing of

sample textbooks for grades seven through twelve. Dr. E. G. Begle was the director of the project which was financed by the National Science Foundation.

In addition, the SMSG has founded another project which will provide the secondary school mathematics teacher and student with supplementary material in the various branches of modern mathematics. This project is known as the New Mathematics Library and includes titles from such mathematical topics as large numbers, inequalities, geometry, and calculus.

Other Study Groups

These three groups by no means exhaust the total number of study groups during this period but they are cited because of their actions which have given leadership to secondary schools in revising their mathematics programs to meet the needs of the times. There are several other national committees and many local, college, and state experiments and study groups which are following the leads of the foregoing projects and expanding upon them to meet situations peculiar to the particular locale.

For example, among other national committees, state, college, and local groups that are revising their mathematics program to meet the needs of the times are:

(1) University of Maryland Mathematics Project. This program was designed to develop improved mathematics programs for grades seven and eight. The original project and its experimentation was confined to schools near the University of Maryland. The courses developed by this program were designed to serve as a bridge between arithmetic and high school mathematics.¹

¹The Revolution in School Mathematics, National Council of Teachers of Mathematics, 1961, p. 19.

(2) Boston College Mathematics Institute. This project is attempting to develop textbooks for mathematics classes grade eight through twelve. The emphasis is on the structure of mathematics approached from the historical point of view.¹

(3) The Greater Cleveland Mathematics Program. This program provided a teacher training program for arithmetic and mathematics teachers of grades one through twelve. The program was primarily devoted to the implementation of new mathematical developments in the classroom. New materials of the School Mathematics Study Group were used.²

(4) Kanawha County Schools, Charleston, West Virginia. This program included grades seven through twelve. The courses began with "Accelerated Arithmetic" and "Elementary Algebra" in grades seven and eight respectively. The course of study terminated with "Trigonometry and College Algebra" in grade eleven and "Mathematical Analysis" in grade twelve.³

MODERN MATHEMATICS IN SELECTED SECONDARY SCHOOL SYSTEMS

Modern mathematics may rightfully be called contemporary mathematics. It is both a point of view and new subject matter. Dr. Albert E. Meder, for example, defines modern mathematics in these words:

... modern mathematics is a point of view, mathematics is thought of as consisting of the deductive development of certain propositions from a list of assumptions accepted without proof about terms undefined except as limited by the conditions of the accepted assumptions. In

¹Loc.cit.

²George Baird, "The Greater Cleveland Mathematics Program," The Mathematics Teacher, January 1961, Vol. 54, No. 1, p. 31.

³B. G. Pauley, "An Evaluation of a Secondary School Mathematics Program for able Pupils," The Mathematics Teacher, May 1961, Vol. 54, No. 5, pp. 324-332.

particular, . . . geometry is the study of invariance and algebra the study of structure.¹

This new point of view is attained by viewing the older "traditional" mathematics in a new perspective. As an example, the traditional point of view considered algebra to be studied as a form of manipulative skill while the modern mathematical point of view places the prime emphasis of instruction on the structure of the system and on deductive thinking. It does not discount the importance of manipulative skills which are necessary for efficient mathematical thought, but rather no longer holds this as the goal of instruction.²

Since the development of mathematics has been continuous, the newer ideas have evolved out of the older ones. There has been no sharp line of division between "traditional" mathematics and "modern" mathematics.

Dr. Meder states further that modern mathematics is also new subject matter. The new point of view has lead to the investigation of many systems of assumptions which were previously neglected. These investigations have, in turn, led to the development of new mathematical knowledge.³

The new developments in mathematics have been extensive. Furthermore, the expanding development of mathematics has resulted in the creation of new subject matter. For example, mathematical research has produced such fields as topology, probability, and statistical inference, modern abstract algebra, and mathematical logic. These developments have led to new courses which graduate students are pursuing in colleges and universities across the land.

¹Albert E. Meder, Modern Mathematics and its Place in the Secondary School, College Entrance Examination Board, 1958, p. 5.

²Program for College Preparatory Mathematics, College Entrance Examination Board, 1959, p. 2.

³Albert E. Meder, loc. cit.

Pure mathematics has experienced spectacular growth and mathematicians have achieved a new and more rewarding place in society. These developments have had important consequences in such fields of applied mathematics as industry, business, the physical sciences, and the social sciences. Thus, the purpose of mathematics is no longer limited to grinding out answers to engineering problems. Also, in its new role, mathematics finds itself producing mathematical models that forecast the outcome of social trends and even the behavior changes of the group.¹

In addition, it appears that modern, or contemporary mathematics possesses certain characteristics which distinguish it from traditional mathematics. Support of this statement may be seen in a recent quotation presented by the College Entrance Examination Board as follows:

... contemporary mathematics is characterized by: (1) a tremendous development quantitatively; (2) the introduction of new content; (3) the reorganization and extension of older content; (4) renewed, increased, and conscious emphasis upon the view that mathematics is concerned with abstract patterns of thought.²

Recent years have witnessed an ever increasing development of mathematical knowledge. During this period mathematical concepts and methods have become mixed and integrated. Mathematicians have pursued these ideas with single-minded devotion. On the other hand, secondary school mathematics teachers have concentrated on the reformulation and presentation of the old traditional ideas. Consequently there has been a definite and prolonged time lag in the communications between the mathematicians and the teachers.³

¹K. E. Brown, "Improved Programs in Mathematics Require Inservice Education for Teachers," The Mathematics Teacher, February 1961, 55:85.

²Program for College Preparatory Mathematics, College Entrance Examination Board, 1959, p. 3.

³A Summary of the Report of the Commission on Mathematics, College Entrance Examination Board, 1959, pp. 10, 11.

It has been noted that colleges and universities have included in their curriculums, courses in the fields of modern mathematics. It has also been noted that the secondary school fails to reflect this change which is so evident in higher educational institutions. Hence, in the present situation there exists many gaps and overlaps in the teaching efforts of these two levels of mathematics education. The continual sequence needed for progress toward desired mathematical competency is often lacking. It is apparent that there is a definite need for better articulation between college mathematics and secondary school mathematics.¹

With the realization that secondary school mathematics is one of the most important training fields for mathematicians, engineers, and scientists, as well as other users of mathematics, the seriousness of the problem is soon visualized. The secondary school holds not only the key to the improvement of mathematics in our colleges and universities, but also the key for increasing the mathematical literacy of this country's populace. In this regard, both traditional and modern mathematics have unusual abilities to stimulate the inquiring mind. Presented in a contemporary spirit modern mathematics should excite and challenge secondary school students.²

It appears evident that the secondary school mathematics program should incorporate the newer mathematical concepts and challenge its students to greater insights and understandings of mathematics. The new secondary school mathematics program must be oriented to the needs of the second half of the twentieth century. Such a program will require greater articulation and

¹Howard H. Fehr, "New Thinking in Mathematical Education," The Mathematics Teacher, October 1960, 53: 424-429.

²A Summary of the Report of the Commission on Mathematics, College Entrance Examination Board, 1959, pp. 6, 7.

understanding between mathematicians, college instructors, and secondary school mathematics teachers. These groups must work together in experimentation and study in order that the goal of a well coordinated modern mathematics program will be achieved.

The secondary school curriculum has been characterized by study and experimentation during the past decade. Accordingly, a part of the study and experimentation has been devoted to the secondary school mathematics program. The need for and the trend of such study and experimentation in the mathematics program of the secondary schools may be seen in the following statement by Myron Rosskoff:

There already exists an aroused public concern with the problem. . . . A period of transition from the older traditional mathematics program to a newer, different sort of organization and content of the mathematics courses offered in the four years of a high school program is now visualized. . . . The routes taken to develop these concepts will be new. They will lie in the areas of contemporary mathematical ideas and thought.¹

The Commission on Mathematics of the College Entrance Examination Board has been a leader in the area of formulating a secondary school mathematics program oriented to present and future needs. Furthermore, the Commission on Mathematics regards the following three principles as fundamental to any such proposed program:

- (1) The proposals must be based on the existing mathematics curriculum and must consist of modification, modernization, and improvement of the present program, rather than its discontinuance and replacement by entirely new content.
- (2) The guide in determining the modifications to be made must be the point of view of modern mathematics rather than that of the traditional

¹Myron Rosskoff, "Articulation of a Secondary School Mathematics Program with College," California Journal of Secondary Education, February 1956, 31:99.

mathematics.

(3) The proposed changes must be sufficiently far reaching such that the new mathematics program is truly oriented to present and future needs, but not so radical as to be beyond the competency of the teaching staff.¹

The new mathematics program of the secondary school must be directed toward the college-capable youth. Support is given to this statement by Howard H. Fehr who wrote:

. . . mathematics studied in the high school has preparatory value not only for college study of pure mathematics, but also for engineering, and applied science. Students need to know series, sequences, probability, solution of formulas and equations, fundamentals of trigonometry, and a knowledge of derivation and integral. Hence, secondary school mathematics must teach concepts, the why and the relations, as well as the skills of operations.²

Further, the Commission on Mathematics of the College Entrance Examination Board has developed a nine point mathematics program for college-capable students. The program is designed to prepare such students with an adequate mathematical background for college work in both pure and applied mathematics. The nine points of the program are:

1. Strong preparation, both in concepts and in skills, for the college mathematics at the level of calculus and analytical geometry.
2. Understanding of the nature and role of deductive reasoning . . . in algebra, as well as geometry.
3. Appreciation of mathematical structure ("patterns") . . . for example, properties on natural, rational, real, and complex numbers.
4. Judicious use of unifying ideas . . . sets, variables, functions, and relations.

¹Modernizing the Mathematics Curriculum, College Entrance Examination Board, 1958, p. 7.

²Howard H. Fehr, Teaching High-School Mathematics, National Educational Association, October 1955, p. 12.

5. Treatment of inequalities along with equations.
6. Incorporation with plane geometry of some coordinate geometry, and essentials of solid geometry and space perception.
7. Introduction in grade 11 of fundamental trigonometry . . . centered on coordinates, vectors, and complex numbers.
8. Emphasis in grade 12 on elementary functions (polynomial, exponential, circular).
9. Recommendation of additional alternative units for grade 12: either introductory probability with statistical applications, or an introduction to modern algebra.¹

The first seven points on the proposed program presented above are essentially modifications to and changes within the already existing program of secondary mathematics. These points emphasize the new point of view of modern mathematics.

With respect to the nine point program previously outlined, it is in points eight and nine that the weight of the new developments regarding mathematical knowledge is felt. These two points emphasize the new content of modern mathematics. It is upon these proposals that secondary school educators can develop a modern mathematics program suitable for its advanced mathematics students and its twelfth grade mathematics students.

Both the Commission on Mathematics of the College Entrance Examination Board and the School Mathematics Study Group have followed the nine points of this program of modern mathematics for college-capable students in preparing experimental textbooks in secondary school mathematics. These textbooks have provided modern mathematics instructional materials for experimental modern mathematics classes in secondary schools.

In addition, the nine point program of the College Entrance Examination

¹Program for College Preparatory Mathematics, College Entrance Examination Board, 1959 p. iii.

Board's Commission on Mathematics has been a guide to mathematics textbooks authors and publishers who have developed textbooks incorporating the spirit and content of modern mathematics. For example, the Charles E. Merrill Book Company has published a geometry textbook in accordance with the recommendation of the sixth point. This development of secondary school mathematics textbooks is in keeping with the spirit of the modern mathematical point of view. The textbook contains an early introduction to solid geometry. This introduction is used in developing both the student's space perception and his understanding of analytical geometry. Further, in the beginning chapters of the text inductive reasoning is discussed. In addition, the topics of plane geometry are related to algebra at every reasonable opportunity.¹

Until the Commission on Mathematics of the College Entrance Examination Board set forth the foregoing nine point program, little, if any, direction was given secondary school officials in the development of a mathematics curriculum suitable for advanced and college-capable students. The local secondary school mathematics programs for such students depended upon the knowledge, foresight, and initiative of local teachers and curriculum planners. Such programs of modern mathematics for advanced and college-capable students appear to have been the exception rather than the rule.

It is evident from the literature that modern mathematics programs have been implemented in some secondary schools. For example, such a program is seen in the Oak Park and River Forest High School of Oak Park, Illinois. This program offered capable students to complete course work in both Plane and Solid Geometry, Algebra II and Trigonometry by the end of their junior

¹Goodwin, Vannetta, and Fawcett, Geometry: A Unified Course, Charles E. Merrill Books, Inc., Columbus 16, Ohio, 1961.

year. In their senior year these students were permitted to enroll in Advanced Mathematics 1 and 11 if they so desired. Each Advanced Mathematics course was one semester in length and covered topics generally reserved for college freshmen.¹

The textbook employed in Advanced Mathematics 1 and 11, in the Oak Park high school, was Fundamentals of College Mathematics by Johnson, McCoy, and O'Neill, Rhinehart and Company, September, 1958. Students studied such topics as the straight line, curve tracing, equation of a locus, translation and rotation of axes from Analytical Geometry. Topics from the Calculus were also studied. These topics included limit and function, differentiation of both algebraic and transcendental functions, and integration of algebraic, trigonometric, and exponential functions.²

The mathematics program for the college-capable student in his senior year of the Oak Park and Forest River High School, closely resembles the mathematics program of most college freshmen who plan to major in mathematics. Upon satisfactory completion of this course students can enroll in some colleges with an advanced standing in mathematics.³

Another secondary school which has developed an advanced mathematics program is the William Penn Senior High School of York, Pennsylvania. Its mathematics course for college-capable students covers the elements of Differential and Integral Calculus. The textbook used in this mathematics course was Differential and Integral Calculus, Vol. 1, by R. Courant, 1957

¹Kenneth W. Lund, "Adapted Syllabus," The Bulletin of the National Association of Secondary School Principals, December 1958, Vol 42, No. 242, pp. 137, 138.

²Loc. cit.

³Loc. cit.

edition. The definite integral, formal integration, the derivative, and formal derivation were the major topics studied by those enrolled in this advanced mathematics course for college-capable students.¹

In other secondary schools, students are trained to operate calculating machines which are widely used today. In such a course knowledge of and operation of calculating machines is emphasized rather than mental calculation.

One such program for advanced mathematics students, in which training in the operations of calculating machines is offered, is that of the Sleepy Hollow High School of North Tarrytown, New York. Students enrolled in the course were instructed in the use and operation of ten-key adding-listing and full-bank adding-listing machines, key-driven calculator, and the Friden, Marchant, and Monroe Rotary Calculators.²

It seems important to emphasize here that the purpose of this program is not to turn out employable machine operators, but to provide these students with an additional tool of learning by qualifying him to competently perform the operations of which each machine was capable and at a reasonable rate of speed.³

In addition, superior mathematics students of the Herbert Hoover Senior High School, San Diego, California, were permitted to take a course in the elements of the digital computer and its allied programing. Students participating in the course, wrote programs for a model computer, ranging from

¹O. Meredith Parry, "Adapted Syllabus," The Bulletin of the National Association of Secondary School Principals, December 1958, Vol. 42, No. 242, pp. 139-142.

²James M. Lynch, Jr., "Of Calculus and Calculators," The Balance Sheet, October 1959, (a reprint).

³Loc. cit.

linear equations to systems of linear equations and quadratics. Each student had the opportunity to be creative and use his ingenuity to produce programs which were useful digital programs for the chosen model computer. In addition to the programming experience received, students also explored vocational opportunities in this field.¹

The Department of Mathematics of the Bronx High School of Science, New York, New York, has developed a differentiated course offering enabling advanced and college-capable students with varying mathematical interests and abilities to pursue mathematics beyond the traditional secondary school mathematics. These mathematics students may enroll in one or more of the following mathematics courses:

- (a) Twelfth-Year Mathematics: This course covers advanced algebra and solid geometry with some vector analysis and solid analytics.
- (b) Twelfth-Year Mathematics: This course is the New York State Experimental course which covers most of "Principles of Mathematics" by Aillendoerfer and Oakly.
- (c) Mathematical Techniques in Science: This course includes a thorough preparation in such fields as "Programming for an Electronic Computer" and gives background in navigation, astronomy, theoretical chemistry, theoretical biology, and theoretical physics.
- (d) Analytical Geometry and the Calculus: This course approximates that of most freshman college mathematics courses of the same name.
- (e) Twelfth-Year Mathematics: This course included the material of (a) and (b) and, in addition, advanced material in programming, statistical inference, and abstract algebra.

The last two courses in the listing above are taken by the most capable mathematics students of the high school. The school has received favorable responses from not only the students and the teachers, but also from colleges

¹Llewel T. Van Tassel, "Digital Computer Programming in High School Classes," The Mathematics Teacher, April 1961, Vol. 54, No. 4, pp. 217-219.

in which those students, who have participated in the course, have enrolled.¹

Until recently, programs such as the afore mentioned depended upon the initiative and direction of the local secondary school's mathematics teacher or Mathematics Department. Course material, therefore, had to be screened and organized from college textbooks. Since this process was largely dependent upon local initiative, such advanced mathematics courses were often unrelated to other such courses in other secondary schools across the nation, as well as infrequent in the American secondary school mathematics program.²

ANALYSIS OF A MODERN MATHEMATICS TEXTBOOK

The preceding section of this study has made limited mention of the use of modern mathematics textbooks in the school systems discussed. Since such textbooks are so important as a teaching tool, in a modern mathematics program, it would seem to be a serious omission if a more extensive treatment of a modern mathematics text was not conducted.

In this regard the text, Advanced High School Mathematics,³ which is one of such texts employed in the advanced mathematics class of the Carbondale Rural High School, was selected for analysis. The authors, in writing this text, followed the recommendations of the College Entrance Examination Board's Commission on Mathematics and the course outlines of the School Mathematics Study Group. Hence, the textbook contains much of the modern mathematics

¹Irving Allen Dodes, "Mathematics at the Bronx High School of Science," The Bulletin of the National Association of Secondary School Principals, May 1959, Vol. 43, No. 247, pp. 81-83.

²Julius H. Hlavaty, "Mathematics in Transition," The Mathematics Teacher, January 1961, pp. 27-30.

³Vannatta, Carnahan, and Fawcett, Advanced High School Mathematics, Charles E. Merrill Books, Inc., Columbus 16, Ohio, 1961.

intermingled with topics traditionally taught to senior mathematics students and its purpose is to bridge the mathematical gap between intermediate algebra and the calculus. In so doing, standard elements of trigonometry, analytical geometry, and college algebra are included in this text.¹

The early chapters of the afore mentioned textbook deal with the real number system and the foundations of algebra, including a consideration of elementary theory of equations. Numeration systems are discussed and applications are given. Also, one section presents students with basic theory and functions of the binary system and its relation to modern electronic computers. Study of such systems aids in developing a better understanding of our own system of numeration.

Further, the textbook provides extensive treatment on the topics of complex numbers and the logical reasoning associated with algebra. The textbook also attempts to encourage students to make generalizations and develop proofs which are simple and logical. The analogies presented in the afore mentioned textbook are kept at a minimum, and basic reasons are employed to strengthen student's conceptual understanding.

Further examination of the text reveals that irrational numbers and the indirect proof are introduced to students at an early stage. In this regard it may be of interest to note that Euclid's indirect proof that $\sqrt{2}$ is irrational is presented in such a readable manner that few students using this textbook should find it difficult to understand. Moreover, the textbook not only provides a summary of the indirect proof which lends itself to simplicity for teaching, but also contains an illustrative model for student use in

¹Ibid., pp. iii and iv.

determining other numbers either rational or irrational.¹

In addition, the textbook employs the modern mathematical point of view in presenting linear functions and their graphical representation in relation to the "set concept." The set concept is a familiar and simple one; i.e., a set of objects consists of all objects which have a common characteristic. The importance of the set concept in modern mathematics is seen in the fact that such branches of mathematics as logic, topology, statistics, and circuit design for electronic computers have been simplified by the introduction of the set concept. It appears important that secondary school mathematics students be taught the traditional mathematics material by means of these newer and more useable concepts. Such teaching would be in harmony with the Commission on Mathematics of the College Entrance Examination Board's fourth principle.

Further analysis of the textbook reveals the inclusion of mathematical tools and concepts such as: (1) synthetic division; (2) curve sketching; (3) iteration; and (4) Newton's method for finding the approximate value of a root. Also included are other evidences common to college and university practice such as methods, symbolism, and definitions.

The importance of "Measurement" as a unifying concept is evidenced in the afore mentioned textbook as one entire chapter is devoted to the subject. In this chapter, the authors emphasize the importance of exact and approximate numbers and measurement is considered in terms of accuracy and precision. This is not surprising since knowledge of such concepts of measurement as absolute error, relative error, and tolerance intervals enables students to

¹Vannatta, Carnahan, and Fawcett, Advanced High School Mathematics, Charles E. Merrill Books, Inc., Columbus 16, Ohio, 1961, pp. 5, 6.

develop greater confidence in the accuracy and precision of their measurements.

In regard to the use of exact and approximate numbers in measurement, the chapter includes an introduction to the concept of significant digits. Further, it emphasizes the importance of significant digits in our present method of measurement in both general and scientific usage. At the end of the chapter the text contains a list of facts about significant digits. The list is presented in summary form for student reference when using approximate numbers with various numbers of significant digits in computation of problems presented in the textbook.¹

In regard to computation with approximate numbers, students are guided by the following principle which the textbook holds fundamental: "The result of computation with approximate numbers cannot be more accurate than the least accurate number that enters into the computation."²

The Advanced High School Mathematics book also introduces the student to linear interpolation, whereby he can estimate the value of a function between two or more known values of the function. In this regard, the procedures are related to logarithms, roots, and natural values of trigonometric functions.

In addition, the textbook provides basic material concerning slide rule computation for both the student and the teacher. It is evident that by supplementing this source with additional materials the teaching of slide rule computation can be an interesting and challenging learning experience for secondary school mathematics students. Further, the approximate computation of slide rules provides another opportunity for developing student comprehension

¹Vannatta, Carnahan, and Fawcett, Advanced High School Mathematics, Charles E. Merrill Books, Inc., Columbus 16, Ohio, 1961, pp. 81, 82.

²Ibid., p. 85.

of significant digits and precision of measurement. The student's competence in slide rule usage should be a valuable tool to him in future college courses or in apprentice engineering and scientific employment where quick approximate computational information is desired.

Chapters V thru VIII of Advanced High School Mathematics are devoted to trigonometry and its applications. The functions presented are defined in terms of a coordinate system, and graphing of the functions is stressed. In addition, applications of trigonometry to practical situations of our society are given and illustrated throughout the chapters.

It is known that such common measures as velocity, acceleration, weight, forces of all kinds, electric voltage, and resistance can be represented by vectors. Hence, examples of trigonometric solutions of problems involving vector addition are provided in the textbook. Such solutions are dependent upon the Law of Sines and the Law of Cosines and it is important to note that the proofs of these two laws are written in a style that should be understandable to secondary school students. Such readability is of great importance since the student's comprehension of trigonometric solutions, with respect to vector addition or subtraction, will provide him with an important problem solving skill more accurate in general, than graphical methods. Furthermore, this problem solving skill will be applicable to applied mathematical fields as well as pure mathematics.

Further analysis of the textbook reveals that the authors of Advanced High School Mathematics have devoted an entire chapter to the treatment of inequalities. This is in harmony with the Commission on Mathematics of the College Entrance Examination Board which has strongly recommended that inequalities be given a more prominent position in the secondary school mathematics program.

With regard to the text's treatment of inequalities, the traditional symbols of inequality are presented and used throughout the text. These symbols of inequality are illustrated and defined in terms of the number line and its orderly sequence. Also, the principles of inequalities are expressed in the form of theorems and axioms. Further, inequalities of one and two variables are presented and methods of solution are given in terms of the number line and the coordinate plane, respectively. Since the study of inequalities provides students with the opportunity to gain greater competency in graphical work, as well as an additional tool of mathematical knowledge; it is apparent that teacher awareness of the meaning and value of inequalities in modern mathematics and its many applications will also be highly important to the students.

In addition to graphical representation of inequalities, the Advanced High School Mathematics textbook contains a short section which relates inequalities to problems involving production of certain items under existing conditions. In this regard, the solution of such problems necessitates the determination of how many or how few of these items can be produced at the greatest percentage of profit or benefit to the producer. The solution scheme is termed linear programming and involves the solution of several inequalities in which the items to be produced are treated as variables and the existing conditions are treated as constants. It appears that the reality of such problems should interest and challenge capable secondary school mathematics students.

Further analysis reveals that the binomial theorem, which is often presented in secondary school algebra textbooks, is developed in the Advanced High School Mathematics chapter entitled "Sequences and Series." In this

chapter, the discussion of sequences and series includes the topics of limit, convergence, and divergence.

With respect to the concept of the limit of a function, it is important to note that this concept has often been reserved for an elementary course in the calculus. In the textbook studied, the use of the limit concept is related to testing series for convergence. In this regard, two tests are presented in the textbook. One test is a comparison test for convergence and the other test is a ratio test for convergence. A teacher may find additional material concerning limits and convergence in most calculus textbooks.

In addition, the above mentioned chapter contains a complete explanation and example of "proof by mathematical induction." Also exercises are listed in order that students may prove stated relations by mathematical induction. In the text, mathematical induction is shown to the student as the type of proof used to prove the assumption derived from the expansion of a binomial $(x + y)^n$ and the general term of this expansion to be a mathematical truth known as the Binomial Theorem. In this regard, it is important to note that the student should not only recognize the importance of proof by mathematical induction, but also the importance that the Binomial Theorem has to mathematics should be recognized.

Two concepts of mathematics which have received a prominent position in applied mathematics in recent years are: (1) Permutations, combinations, and probability; and (2) Descriptive statistics. The prominence of these concepts evolves from their importance as tools of modern business, educational, and scientific mathematics needs. Moreover, most aspects of twentieth century life can and do use these two mathematical concepts in some manner.

Since all secondary school students will be required to choose from an aspect of life involving the above mentioned mathematical concepts, it appears

important that such students obtain both an understanding of and an appreciation of these concepts.

The concept of permutations, combinations, and probability includes the determination of possible arrangements of given events, the determination of the probability of events occurring at a designated time or place, and the determination of possible choices from a group of items. Students mastering this concept should develop a clearer and more profound understanding of mathematical logic and mathematical certainty as related to their lives and the events which take place in this world.

The textbook reveals the relationship between probability and the Binomial Theorem to both teacher and student. In this regard, the textbook introduces the binomial experiment and students are given the opportunity to learn how to use the binomial experiment. The binomial experiment is an experiment in which the number of trials is fixed and the trials are independent. In such an experiment it is the number of successes which is important rather than the order of success. The probability of the success in a given event is constant and every event must end in either success or failure. In addition, several illustrative examples are presented in the textbook to enhance the student's understanding of these concepts. Moreover, the concept of probability and its relation to the binomial theorem should not only strengthen the student's competency and understanding of algebra, probability, and related concepts, but should also prepare these students for the study of statistics.¹

It has been recommended by the Commission on Mathematics of the College Entrance Examination Board that secondary school students have an opportunity

¹Vannatta, Carnahan, and Fawcett, Advanced High School Mathematics, Charles E. Merrill Books, Inc., Columbus 16, Ohio, 1961, pp. 245-251.

to take units in introductory probability with statistical applications. In this regard, the authors of Advanced High School Mathematics have followed the recommendations and guidance of these leaders and have included a chapter entitled "Descriptive Statistics."

This chapter introduces the following concepts to the secondary school mathematics student: measures of central tendency, measures of variability, frequency distribution, normal distribution, curve, the standard deviation, sampling techniques, standard error, and related computational methods and graphical representation. As students master these concepts they should also develop skill and understanding in the basic fundamentals of statistics. This knowledge should prepare the student for college courses in statistics, as well as the ability to understand and interpret the general information provided by the many statistical reports which are widely used in today's society.

The latter one-third of Advanced High School Mathematics is devoted to a concise treatment of analytical geometry. Analytical geometry is that branch of mathematics which permits the study of geometrical figures by analytical, or algebraic methods. It also permits geometrical representation of equations with two variables in a coordinate plane. This portion of the textbook considers the concepts of the straight line, the circle, the parabola, the ellipse, and the hyperbola.¹

With respect to the above mentioned concepts, each is carefully defined and described with illustrations provided to avoid misunderstandings. Also, each concept is treated in terms of polar coordinates, as well as in terms of Cartesian coordinates which are generally more familiar to the secondary school

¹Vannatta, Carnahan, and Fawcett, Advanced High School Mathematics, Charles E. Merrill Books, Inc., Columbus 16, Ohio, 1961. pp. 300-377.

mathematics student. The material presented in this portion of the textbook studied is closely correlated to the material covered in college and university analytical geometry courses. Students who satisfactorily complete this course in analytical geometry should be prepared for the calculus.

IMPLICATIONS OF MODERN MATHEMATICS TO THE SECONDARY SCHOOL AND THE MATHEMATICS TEACHER

It has been noted that publishers are now publishing mathematics textbooks which are suited for secondary school mathematics students. They are written in a style which is directed to the capabilities of the secondary school students rather than the capabilities of college and university students or pure mathematicians. The content has been carefully selected such that it will be meaningful and arranged such that it is developed in an orderly sequence.

In addition, leading mathematicians have recently fulfilled the responsibility of determining the nature of college preparatory mathematics courses. With the aid and advice of classroom teachers, mathematics educators, learning specialists and psychologists, they have described patterns of reasoning that characterize mathematics thought. They have set forth the implications of the expanding growth and varied applications of contemporary mathematics in terms appropriate for each grade level.¹

Therefore, it appears that the greatest present implication to the secondary school which is initiating a new program of modern mathematics, is to find a competent teaching staff. In this regard, the satisfactory teacher of secondary school mathematics of such a staff must know a great deal of mathematics.

¹Frank B. Allen, Chapter VI, The Revolution in School Mathematics, National Council of Teachers of Mathematics, 1961. pp. 84-86.

In addition, he must teach mathematics with interest and enthusiasm; thus preserving and strengthening the student's native interest in and enthusiasm for mathematics.

The modern mathematics teacher needs to have greater variance and insight of his subject matter than ever before. Many of the present teachers are inadequately prepared to teach the modern mathematics. Members of the School Mathematics Study Group and members of the Commission on Mathematics of the College Entrance Examination Board are well aware of these two facts. They have recommended that teachers of modern secondary school mathematics receive training in two areas. These two areas are: (1) knowledge of the new mathematics content and the techniques for teaching it, and (2) training in teaching the new approaches to the familiar or traditional mathematics content. It is apparent that additional education and training of mathematics teachers is imperative to the American secondary school system if it is to have an adequate program of modern mathematics for its students.¹

The improvement of our secondary schools' mathematics program for college-capable students is also dependent upon the type of cooperation and articulation among our nation's college and university professors, secondary school teachers, college and university officials, and secondary school administrators. These four groups must work together to provide a well planned and well correlated program of modern mathematics for college-capable students. There are various examples of cooperative programs in which these four groups have worked together in a well-organized manner toward their common goal.

One of the most notable of these cooperative programs is the University

¹W. Eugene Ferguson, "Implementing a New Mathematics Program in Your School," The Revolution in School Mathematics, 1961. pp. 39,40.

of Illinois Curriculum Study in Mathematics. Since 1950 the University of Illinois has been working toward the goal of developing a well coordinated mathematics program for students of grades one through twelve. In the recent years this university has offered a summer school session to secondary mathematics teachers such that they can learn the methods and recommended procedures of teaching these mathematics courses. In addition, the University of Illinois has cooperated with local school systems in providing an in-service training program for teachers using UICSM materials in their mathematics classes. The in-service training program provides teachers with an opportunity to discuss the content and teaching techniques with leading educators and mathematicians while they continue their teaching positions. Such programs provide an avenue for teachers to solve their teaching problems more readily and make effective applications to the classroom more quickly.

This strong program of experimentation, development, and cooperation in elementary and secondary mathematics under the leadership of the University of Illinois has been both an incentive and a guide to other universities and colleges which now have similar programs in operation. Among these other schools which have their own individual projects are the university of Maryland (The University of Maryland Mathematics Project), Boston College (The Boston College Mathematics Institute), Ball State of Muncie, Indiana, (The Ball State Teachers College Experimental Program), and Southern Illinois University (The Developmental Project in Secondary Mathematics). Each of these projects are similar in the fact that they emphasize revision of existing mathematics content to include elements of modern mathematics with a greater stress on the structure of mathematics and the precision of language usage. In addition, each project is characterized by the development of materials to be written into a textbook which may later be utilized by the American secondary

school system in its mathematics classrooms.

Another factor which is helping to prepare the teacher of modern mathematics is the Summer Institutes. These institutes are sponsored jointly by the National Science Foundation and various colleges and universities. Participants receive stipends which cover their tuition and general expenses. Summer Institutes in Mathematics offer courses in four areas. These four areas are: (1) modern mathematical concepts, (2) review of traditional mathematical concepts, (3) content of newly developed mathematics textbooks, suggested courses, and curriculum development for the secondary school and its mathematics program, and (4) methods and techniques for teaching both the traditional mathematics and the modern mathematics. It is readily realized that such institutes are designed to give participants a better foundation for handling the new mathematics curricular innovations which are now in the process of national development.

It appears evident that our nation's colleges, universities, and educational foundations consider the training of mathematics teachers imperative to the teaching of modern mathematics in the secondary school. Hence, the implication to secondary school systems is that they should strongly recommend and encourage their teachers of modern mathematics to enroll in college mathematics courses in one or more of the above mentioned areas.

It has been previously noted that well-qualified teachers of mathematics are needed if a modern mathematics program in the secondary school is to be successful. In this regard, and in light of shortage of secondary school teachers, there appears to exist another implication to the secondary school system with respect to its ability to provide a modern mathematics program taught by competent mathematics teachers.

This implication implies that the American secondary school system

reduce the number of secondary schools with small enrollments. The reason for this reduction is that such schools generally cannot provide their students with a mathematics program which would include concepts and courses noted in this report, as well as provide capable and competent teachers to teach these same concepts and courses. With respect to the afore mentioned implication, it appears that the nation's small secondary school systems should strongly consider consolidation into larger secondary school systems which could more adequately provide ample classrooms, competent mathematics teachers, and other needed mathematics facilities to meet society's present needs in modern mathematics education.

SUMMARY

The summary statements in connection with this study are drawn from analysis of modern mathematics textbooks, from practical applications in secondary school mathematics programs, and from evidence furnished by other studies on modern mathematics for the secondary school. It has been concerned with the modern mathematics in the secondary school mathematics program; considering the content and practices of such programs. In addition the study has been concerned with finding any implications such modern mathematics programs may have to the secondary schools and to the members of the mathematics teaching staff.

It has been noted that modern mathematics includes both new mathematical content and a new point of view concerning older or traditional mathematical content. Only in recent years, have these concepts of modern mathematics been given a prominent position in secondary school mathematics programs.

The recent years have witnessed mathematicians and educators of every level working together to provide an improved mathematics program for the

secondary school. As noted in this study, both the Commission on Mathematics of the College Entrance Examination Board and the School Mathematics Study Group have been composed of such members and have worked toward this same goal of an improved mathematics program for the secondary school. These groups have set forth not only principles and guidelines for the secondary school to follow; but also new mathematics materials in which they present some of the implications of the tremendous growth and many applications of twentieth century modern mathematics.

In addition to these groups, many educators, writers, and mathematicians are collaborating in writing modern mathematics textbooks especially for the secondary school mathematics program and its students. In this regard, the textbook, Advanced High School Mathematics, was analyzed and found to include mathematical concepts and ideas which are in harmony with the new developments in modern mathematics for the secondary school mathematics student. Hence, textbooks, such as this one, will provide the secondary school mathematics teacher with instructional material which reflects the recommendations of the Commission on Mathematics of the College Entrance Examination Board with regard to the secondary school student's mathematics comprehension. The importance of developing a mathematics textbook in the light of such major goals is highly desirable since the comprehension of mathematical operations as well as comprehension of mathematical concepts is of paramount importance to the student's mathematical learning not only at the secondary school level but also if the student is to further his mathematical education in college.

As the secondary school strengthens its mathematics program there will develop a need for more textbooks which are devoted to concepts which the authors of Advanced High School Mathematics have treated in one or just a few chapters. It is conceivable that textbooks will be developed for each of such

mathematical concepts as statistics, probability, analytical geometry, and the calculus. To illustrate, the Commission on Mathematics of the College Entrance Examination Board has printed a full semester course outline in probability. In addition, the School Mathematics Study Group has printed sample editions of textbooks in the areas of statistics and elementary functions. These materials provide secondary school systems with full semester courses of study in these concepts.

Statistics, probability, analytical geometry, and the calculus are the concepts of contemporary mathematics which are now being given and will have a greater degree of importance in secondary school mathematics instruction. In addition, another area of modern mathematics that is gaining importance in the college and university mathematics program, and may soon find its place in secondary school mathematics classrooms, is linear programming. Already, experimental programs in linear programming and its related use in modern electronic computers have experienced success at the secondary school level.¹

Students mastering the above mentioned concepts should be prepared to carry on their mathematical training in college and university mathematics courses with a high degree of success. With improved secondary school mathematics programs, many students will have opportunity to attain higher levels of mathematical literacy during their college and university years. For many of these students, this means more effective study of science, engineering, and many other subjects or an earlier access to the frontiers of mathematical thought and longer careers as creative mathematicians. This is important because mathematicians are now in short supply, and mathematicians, like

¹Darrel G. Littlefield, "Computer Programming for High Schools and Junior Colleges," The Mathematics Teacher, April 1961, 54:220-223.

athletes, reach their maximum efficiency early in life.

The teacher's role with respect to the secondary school program of modern mathematics is quite clear. The mathematics teacher must retain that which was good in the traditional programs, evaluate the new concepts from the standpoint of teachability, and, most important of all, transform the new theories of modern mathematics into action in the classroom. As concepts of modern mathematics which at first seem unteachable are found on closer analysis to be appropriate for study by high school mathematics students, the secondary school mathematics teacher must adequately prepare himself by continuing his mathematics education and include college courses in modern mathematics.

With regard to the mathematics teacher continuing his mathematics education, this study has indicated some of the possible ways the mathematics teacher may do so. The three most notable ways are in-service training, summer school, or a full academic year of study.

The major implication to the American secondary school system, as a whole, appears to be the need for the reduction of secondary schools with small enrollments. It has been found that such schools generally cannot provide their students with a mathematical program which would include the concepts and courses noted in this report. This indicates that our nation's small school systems should consolidate into larger school systems which can adequately provide ample classrooms, teachers, and facilities to meet society's present and predictable future needs in modern mathematics education.

In conclusion, it appears that the American secondary school and interested groups of mathematicians and educators have made significant steps in both the development and the implementation of a mathematics program which will provide adequate modern mathematics education for its students. Society, its secondary school, its colleges and universities, its mathematicians, and its students must continue to work together toward this goal.

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REFERENCES

Books

- Alexander & Saylor, Modern Mathematics Education, Rinehart & Co., Inc., New York, 1959.
- Campbell & Caswell, Readings in Curriculum Development, Houghton & Mifflin Co., New York, 1937.
- Conant, James B., The American High School Today, McGraw-Hill Book Co., Inc., New York.
- Goodwin, Vannatta, & Fawcett, Geometry--A Unified Course, Charles E. Merrill Books, Inc., Columbus 16, Ohio, 1961.
- Kinney & Purdey, Teaching of Mathematics in the Secondary Schools, Rinehart and Co., Inc., New York, 1958.
- Vannatta, Carnahan, & Fawcett, Advanced High School Mathematics, Charles E. Merrill Books, Inc., Columbus 16, Ohio, 1961.

Periodicals

- Baird, George, "The Greater Cleveland Mathematics Program," The Mathematics Teacher, 54: 31, January 1961.
- Brown, K. E., "Improved Programs in Mathematics Require Inservice Education for Teachers," The Mathematics Teacher, 54: 85-89.
- Dodes, Irving Allen, "Mathematics at the Bronx High School of Science," The Bulletin of the National Association of Secondary School Principals, 43: 81-83, May 1959.
- Douglas, H. R., "Lets Face the Facts," The Mathematics Teacher, February 1937.
- Fawcett, H. P., "Guidelines in Mathematics Education," The Mathematics Teacher, 53: 418-23, October 1960.
- Fehr, H. H., "New Thinking in Mathematics Education," The Mathematics Teacher, 53: 424-29, October 1960.
- Grossman, George, "Probability and Statistics in the Twelfth Year," The Mathematics Teacher, 54: 540-46, November 1961.
- Hale, William T., "UICSM's Decade of Experimentation," The Mathematics Teacher, 54: 613-18, December 1961.
- Halavaty, Julius H., "Mathematics in Transition," The Mathematics Teacher, 54: 27-30, January 1961.

- Littlefield, D. G., "Computer Programming for High Schools and Junior Colleges," The Mathematics Teacher, 54: 220-23, April 1961.
- Lund, Kenneth W., "Adapted Syllabus," The Bulletin of the National Association of Secondary School Principals, 43: 137,38, December 1958.
- Lynch, James M., Jr., "Of Calculus and Calculators," The Balance Sheet, (a reprint), October 1959.
- Parry, O. Meredith, "Adapted Syllabus," The Bulletin of the National Association of Secondary School Principals, 42: 139-42, December 1958.
- Pauley, B. G., "An Evaluation of a Secondary School Mathematics Program for Able Pupils," The Mathematics Teacher, 54: 324-32, May 1961.
- "Recommendations of the Mathematical Association of America for the Training of Teachers of Mathematics," The Mathematics Teacher, 53: 632-38, December 1960.
- Roszkoff, Myron, "Articulation of Secondary School Mathematics Program," California Journal of Secondary Education, 31: 99-100, February 1956.
- Schumaker, John A., "Trends in the Education of Secondary School Mathematics Teachers," The Mathematics Teacher, 54: 413-22, October 1961.
- Taylor, Angus E., "Convention and Revolt in Mathematics," The Mathematics Teacher, 55: 2-9, January 1962.
- Van Tassel, Lowell T., "Digital Computer Programming in High School Classes," The Mathematics Teacher, 54: 217-19, April 1961.
- Wagner, John, "The Objectives and Activities of the SMSG," The Mathematics Teacher, 53: 454-59, October 1960.
- Williams, H. E., "The Place of Probability and Statistics in the Mathematics Curriculum of the Senior High School and Junior College," The Mathematics Teacher, 54: 316-20, May 1961.

Publications of Learned Organizations

- Allen, Frank B., "Chapter VI," The Revolution in School Mathematics, National Council of Teachers of Mathematics, 1961, pp. 84-86.
- A Summary of the Report of the Commission on Mathematics, The College Entrance Examination Board, 1959.
- Fehr, Howard H., Teaching of High-school Mathematics, National Council of Teachers of Mathematics, October 1955.
- Ferguson, W. Eugene, "Implementing a New Mathematics Program in Your School," The Revolution in School Mathematics, National Council of Teachers of Mathematics, 1961, pp. 37-51.

Meder, Albert E., Modern Mathematics and its Place in the Secondary School, The College Entrance Examination Board, 1958, 13 pp.

Modernizing the Mathematics Curriculum, The College Entrance Examination Board, 1958, 16 pp.

Program for College Preparatory Mathematics, The College Entrance Examination Board, 1959.

Sets, Relations, and Functions, The College Entrance Examination Board, 1958, 35 pp.

The Revolution in School Mathematics, National Council of Teachers of Mathematics, 1961, 90 pp.

MODERN MATHEMATICS
IN THE SECONDARY SCHOOL

by

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B. S. Bethany College, Lindsberg, Kansas, 1960

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The threefold purpose of this study was to (1) discover what modern mathematics is; (2) review selected modern mathematical teaching materials for secondary school mathematics programs; and (3) discover what implications modern mathematics holds for the secondary school's mathematics programs and the secondary school mathematics teacher.

The procedure employed in this study was to review the pertinent literature contained in the Kansas State University library and five modern mathematics textbooks received from publishing companies. This investigation produced a considerable amount of information pertaining to modern mathematics in the secondary school which was organized into the following categories for presentation:

1. Development of mathematics at the secondary level in the American educational system.
2. Development of mathematics in secondary schools during the decade 1950-60.
3. Instructional materials developed and written by national study groups devoted to modern mathematics in the secondary school.
4. A textbook devoted to modern mathematics written for the secondary school.

It was found that modern mathematics includes both new mathematical content and a new point of view concerning older or traditional mathematical content. Only in recent years, have these concepts of modern mathematics been given a prominent position in secondary school mathematics programs.

In addition, the recent years have witnessed mathematicians and educators of every level working together to provide an improved mathematics programs for the secondary school. These groups have set forth not only principles and guidelines for the secondary school to follow; but also new

mathematics materials in which they present some of the implications of the tremendous growth and many applications of twentieth century modern mathematics.

In regard to textbooks, Advanced High School Mathematics was reviewed and found to contain many of the topics which are considered as modern mathematics and was written in a style easily read and understood by secondary students. It is conceivable that similar textbooks will be developed for each of such mathematical concepts as statistics, probability, analytical geometry, and the calculus.

With regard to the teacher of mathematics it was discovered that he needed better preparation in order to teach modern mathematics effectively and competently.

In conclusion, it appears that the American secondary school and interested groups of mathematicians and educators have made significant steps in both the development and the implimentation of a mathematics program which will provide adequate modern mathematics education for its students. Society, its secondary school, its colleges and universities, its mathematicians and its educators must continue to work together toward this goal.